Closed-Form Admittance Calculation for Generalized Periodic SAW Transducers

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Abstract

Analytic formulae for surface acoustic wave (SAW) transducer admittance calculation comprising both acoustic radiation conductance and susceptance are deduced neglecting multiple interelectrode SAW interactions (quasi-static approximation). Within model constraints applied, an acoustic admittance of the aperture-weighted SAW transducer is treated as a weighted sum of the nodal interelectrode admittances. By applying a special summation technique for the apodized periodic SAW transducers with a fixed pitch and metallization ratio the general formula is converted to the compact reduced form resulting in considerable reduction of the computation time if compared to the wide-spread aperture channelizing technique.

Introduction

Calculation of a SAW transducer admittance is an integral part of the computer-aided design of SAW bandpass filters. Accurate modelling is necessary to predict a priori SAW filter insertion loss, simulate frequency response distortion due to the electrical interaction with source and load, match perfectly SAW devices. Unfortunately, rigorous analysis techniques [1] are impractible due to their intrinsic complexity and computation slowness. Usually, an equivalent circuit model of the uniform SAW transducer is used [2] and aperture channelizing technique [3,4] is applied for apodized SAW transducers. As a rule, it is the radiation conductance that is deduced within SAW transducer model and generally numeric Hilbert transformation is performed to calculate the radiation susceptance [3].

Calculations are simplified in quasi-static approximation [5] where superposition principle can be effectively applied to calculate radiation conductance of an unapodized periodic SAW transducer with arbitrary polarity sequence [3, 4]. Unfortuantely, simple analityc formulae comprising both radiation conductance and susceptance were deduced for uniform multielectrode transducers only [5].

In the present paper closed-form expressions for admittance calculation comprising both acoustic conductance and susceptance are deduced in quasi-static approximation using nodal admittance matrix of a SAW transducer. Within model constraints applied, the acoustic admittance of an aperture-weighted (apodized) SAW transducer is treated as a weighted sum of the elemental interelectrode admittances, with the weights given by the overlaps (partial apertures) of the i-th and k-th fingers. The nodal admittance matrix takes the simplest form for periodic SAW transducers with a fixed pitch and metallization ratio and the general formula for apodized periodic SAW transducers may converted to the compact reduced form by applying a special summation technique and taking into account the periodic properties of the nodal matrix.

Nodal admittance matrix of a SAW transducer

It can be shown using superposition principle that electrode currents I_i and voltages V_k are interrelated via nodal admittance matrix with elements Y_{ik} as follows

$$\mathbf{I}=\mathbf{Y}\,\mathbf{V} \qquad (1)$$

where **I** is the vector of the electrode currents, **V** is the vector of the electrode voltages, and **Y** is the square nodal admittance matrix of size N, with N being the electrode number of a SAW transducer. Due to the reciprocity property the nodal admittance matrix is symmetrical $Y_{ik}=Y_{ki}$ and due to causality principle for voltages and currents the real and imaginary parts of each element Y_{ik} are interrelated via a Hilbert transformation [5].

The nodal matrix elements Y_{ik} are defined as

$$Y_{ik}(\omega)=I_i(\omega)/V_k\,, \quad V_i=0\,, \quad i\neq k \qquad (2)$$

where $I_i(\omega)$ is the current induced in the ith electrode when only kth electrode is activated by applying the voltage V_k, with all the others being grounded (Fig.1).

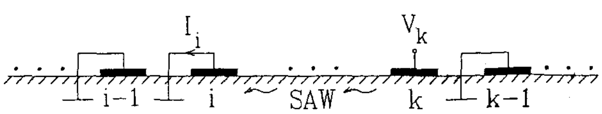


Fig. 1. Acoustoelectric interaction of the ith and kth electrodes in the elemental SAW transducer

In general case the current $I_i(\omega)$ contains both the electrostatic component and acoustic components. The electrostatic component $j\omega Q_i$ where Q_i is the electrostatic charge on the electrode can be found from the solution of the electrostatic problem and contributes to the transducer static capacitance. Closed-form charge and capacitance calculation for the case of the periodic SAW transducers can be found elsewhere [6] and is not considered here. Therefore, our primary interest concerns calculation of the acoustic component induced by the inicident acoustic wave generated by the kth electrode omitting the electrostatic component contribution.

In quasi-static approximation [5] the short-circuit current $I_i(\omega)$ induced acoustically in the ith electrode of the aperture W by the kth electrode with the voltage V_k can be written in the form

$$I_i(\omega)=\omega W\Gamma\, \rho_i^*(\omega)\rho_k(\omega)V_k e^{-j\beta l_{ik}},\ i<k \qquad (3)$$

where $\Gamma=K^2/2\epsilon$ is the piezoelectric constant, with K^2 being the electromechanical coupling factor and ϵ being the substrate permittivity, $\beta=\omega/v$ is the SAW wavenumber, l_{ik} is the separation between ith and kth electrodes. The function $\rho_k(\omega)$ is the Fourier transform of the electrostatic charge density distribution [5] in the elemental SAW transducer structure where the voltage V_k is applied to kth electrode only, with all the others being grounded.

For a periodic SAW transducer with a fixed pitch d and it follows [5] that $\rho_i^*(\omega)=\rho_k(\omega)=\rho(\omega)$ where the function $\rho(\omega)$ known as the element factor is given by (Eq. 4.88, [5]). Thus, the interelectrode admittances $Y_{ik}(\omega)$ of the nodal matrix defined per unit of the overall aperture are given by the following equation

$$Y_{ik}(\omega)=\omega\Gamma\rho^2(\omega)e^{-j|i-k|\varphi},\quad \varphi=\beta d,\quad i\neq k \qquad (4)$$

According to Eq. 4 the matrix elements depend on the indexes difference, i.e. $Y_{ik}(\omega)=Y_{|i-k|}(\omega)$ that allows to introduce one-dimension indexation of the matrix elements $p=|i-k|$. Hence, the overall nodal admittance matrix $\mathbf{Y}$ contains N different elements $Y_{|i-k|}(\omega)$ only, with each sequential row (column) derived by the cyclic shift of the preceding one.
Elements $Y_p(\omega)=-Y_{|i-k|}(\omega)$, $p=0,1,\ldots$ may be treated as interelectrode elemental admittances between the nearest electrodes, next nearest ones, etc. (Fig. 2)

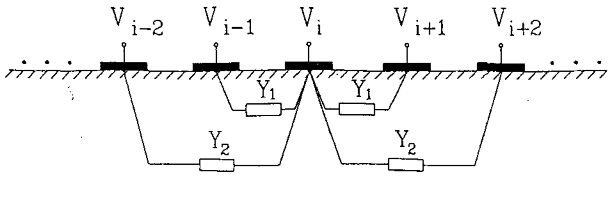


Fig. 2. Self- and interelectrode admittances connected to the ith electrode

Thus, the nodal admittance matrix $\mathbf{Y}$ has the following structure

$$\mathbf{Y}=-\begin{bmatrix} Y_0(\omega) & Y_1(\omega) & Y_2(\omega) & \ldots & Y_{N-1}(\omega) \\ Y_1(\omega) & Y_0(\omega) & Y_1(\omega) & \ldots & Y_{N-2}(\omega) \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ Y_{N-2}(\omega) & Y_{N-3}(\omega) & \ldots & Y_0(\omega) & Y_1(\omega) \\ Y_{N-1}(\omega) & Y_{N-2}(\omega) & \ldots & Y_1(\omega) & Y_0(\omega) \end{bmatrix} \qquad (5)$$

As the consequence of the Kirchhoff's law we could also predict

$$\lim_{N\to\infty}\sum_{i=0}^{N-1} Y_{ik}(\omega)=\lim_{N\to\infty}\sum_{k=0}^{N-1} Y_{ik}(\omega)=0,\quad \omega\neq 0 \qquad (6)$$

where infinite summation is required to account for correctly all electrode currents in the theoretically infinite basic periodic structure [5]. If the electrode number N is large enough we can assume with sufficient accuracy that

$$\sum_{p=0}^{N-1} Y_p(\omega)=0 \qquad (7)$$

It can be shown by analytic integration that the conductance and susceptance components of the off-diagonal elements $Y_p(\omega)$, $p\neq 0$ are interrelated via the Hilbert transformation.

By substituting $i=k$ ($p=0$) into Eq. 4 one can also calculate the conductance of the diagonal self-admittance $Y_{ii}(\omega)$. It is worthy noting that Eq. 4 gives zero value of the self-susceptance that is the direct consequence of the quasi-static approximation applied.

Thus, the nodal admittance matrix for the periodic SAW transducer is totally determined.

Admittance of an unapodized SAW transducer

Given the nodal admittance matrix $\mathbf{Y}$ and SAW transducer electrode voltages $\mathbf{V}$ we can deduce an analytic expression for the transducer admittance from the following considerations. In quasi-static approximation total acoustic power radiated and stored within transducer of the unit aperture $W=1$ is given by the following expression

$$\frac{1}{2}\mathbf{V}^*\mathbf{I}=\frac{1}{2}Y(\omega)\Delta V^2 \qquad (8)$$

where $Y(\omega)$ is the transducer acoustic admittance, ΔV is the voltage applied to the transducer bus-bars, $\mathbf{V}$ and $\mathbf{I}$

are vectors of the electrode voltages and currents, and the asterisk denotes Hermitian conjugation. By taking into account Eq. 1 we can derive the following matrix expression

$$Y(\omega)=\frac{1}{\Delta V^2}\mathbf{V}^*\mathbf{I}=\frac{1}{\Delta V^2}\mathbf{V}^*\mathbf{Y}\,\mathbf{V} \qquad (9)$$

or in the scalar form

$$Y(\omega)=\frac{1}{\Delta V^2}\sum_{i=0}^{N-1}\sum_{k=0}^{N-1}Y_{ik}(\omega)V_i V_k \qquad (10)$$

According to the Eq. 10 the transducer admittance is defined by the weighted sum of the elemental admittances $Y_{ik}(\omega)$. If each electrode is connected to either of two bus-bars with the potentials $\pm\Delta V/2$ then weights take the value +1 for all the equipotential electrode pairs and -1 otherwise.

The known expression for the acoustic conductance [5, Eq. 4.105]

$$G(\omega)=\frac{1}{\Delta V^2}\omega W\Gamma\rho^2(\omega)\left|\sum_{k=0}^{N-1}V_k e^{-jk\varphi}\right|^2 \qquad (11)$$

follows as the real part of the Eq. 10 after substituting Eq. 5 and using the evident identity

$$\left|\sum_{k=0}^{N-1}V_k e^{-jk\varphi}\right|^2=\left(\sum_{i=0}^{N-1}V_i e^{-jk\varphi}\right)\left(\sum_{k=0}^{N-1}V_k e^{-ji\varphi}\right)^*=\sum_{i=0}^{N-1}\sum_{k=0}^{N-1}V_i V_k\cos(i-k)\varphi \qquad (12)$$

However, the Eq. 10 allows to calculate simultaneously both the conductance $G(\omega)$ and and susceptance $B(\omega)$ of the transducer admittance $Y(\omega)=G(\omega)+jB(\omega)$.

Admittance of an apodized SAW transducer

The general formula (10) can be applied to the arbitrary intersection y of the apodized SAW transducer (Fig. 3.) to calculate its contribution $Y(\omega,y)$ to the transducer admittance:

$$Y(\omega,y)=\frac{1}{\Delta V^2}\sum_{i=0}^{N-1}\sum_{k=0}^{N-1}Y_{ik}(\omega)\,V_i(y)\,V_k(y) \qquad (13)$$

where $V_i(y)$ is the voltage on the ith electrode at the intersection y. The total admittance $Y(\omega)$ of the apodized SAW transducer can be found by integrating Eq. 13 over the aperture. By taking into account the property (7) we obtain the following expression

$$Y(\omega)=\int_{-W/2}^{+W/2}Y(\omega,y)\,dy=-\sum_{i=0}^{N-1}\sum_{k=0}^{N-1}W_{ik}Y_{ik}(\omega) \qquad (14)$$

where quantities $W_{ik}=|y_i-y_k|$ are overlaps (partial apertures) of the ith and kth electrodes with the coordinates of the transversal gaps y_i and y_k respectively (Fig. 3).

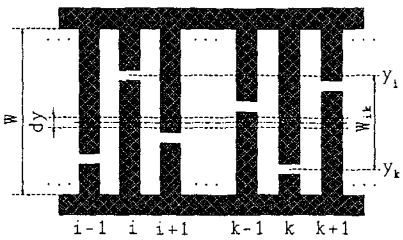


Fig. 3. Elemental intersection of an apodized SAW transducer and partial aperture of ith and kth electrodes

Thus, the admittance of the apodized SAW transducer is the weighted sum of the elemental nodal admittances, with the weights defined by the partial apertures W_{ik}.

Amount of calculations can be considerably reduced by taking into account the property (7) and cyclic structure (5) of the nodal admittance matrix **Y** having N different elements only. By reordering the summation and subsititing $Y_p(\omega)=-Y_{|i-k|}(\omega)$ the Eq. 14 can be converted to the following compact form

$$Y(\omega)=\sum_{p=1}^{N-1}L_p Y_p(\omega) \qquad (15)$$

where

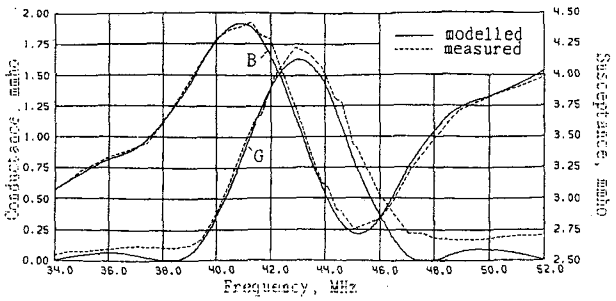
$$L_p=\sum_{k=0}^{N-p-1}W_{k,k+p} \quad or \quad L_p=\sum_{k=p}^{N-1}W_{k,k-p}$$

Quantities L_p are effective apertures defined by the total overlaps of all the nearest, next nearest neighbour electrodes, and so on, respectively. They depend on the SAW transducer apodization only and do not depend on the frequency. Elemental admittances $Y_p(\omega)$ are calculated using Eq. 4. Therefore, the transducer admittance $Y(\omega)$ comprising both conductance and susceptance is essentially defined by the Fourier transform of the effective apertures L_p.

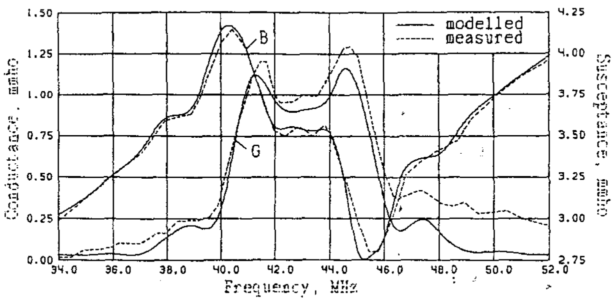
Once the quantities L_p have been calculated and stored in the memory the transducer admittance calculation comprising both conductance and suspectance at any frequency by applying Eq. 15 takes no more time than calculation of the transducer frequency response. Fast Fourier transform can be effectively applied to calculate the admittance characteristic in the wide frequency range.

Calculation example and experimental results

An example of modelled and measured admittance characteristics for a SAW bandpass filter are shown in Fig. 4.



a) input unapodized transducer



b) output apodized transducer

Fig. 4. Modelled and measured admittance characteristics of a SAW bandpass filter

The SAW filter has the folowing parameters: central frequency $f_0=42.7$ MHz, fractional bandpass width (at the - 3 dB level) 10 %, and shape factor 2 at the -3 and -40 dB levels. The filter contains one unapodized and another apodized SAW transducers, both with splitted electrodes. The transducers has an aperture W=2.35 mm and metallization ratio 0.5. Electrode numbers are $N_1=38$ and $N_2=118$ respectively. The substrate material is the YZ-cut lithium niobate with electromechanical coupling factor $K^2=4.5\%$ and permittivity $\epsilon=55\epsilon_0$.

Input and output transdurer capacitances were calculated by applying the technique [6]. Both unapodized and apodized SAW transducer admittances were calculating using the Eq. 15.

There is a good agreement between modelled and measured characteristics in the wide frequency range. The predicted insertion loss value of -19.3 db agrees well with the measured value of -20 dB.

Conclusion

The general formula for calculation of the admittance of the apodized SAW transducers has been deduced in quasi-static approximation using nodal admittance matrix of a SAW transducer. By applying a special summation technique for periodic SAW transducers the general formula may be converted to the compact form resulting in considerable reduction of the computation time if compared to the wide-spread aperture channelizing technique commonly used. According to this formula, an acoustic admittance is defined by the Fourier transform of the effective apertures, with effective apertures values being the total overlaps of all the nearest neighbour fingers, next nearest ones, and so on, respectively. Effective apertures are uniquely defined by finger overlaps and do not depend on the frequency. Assumed for the set of the effective apertures to be determined a priori, acoustic admittance calculation comprising both real and imaginary components takes no more time than frequency response calculation.

The method is quite general and may be applied to capacitively-weighted, polarity-weighted, multi-phase, and other periodic SAW transducers having the central frequency away from the synchronous frequency.

Results of admittance calculation for SAW transducers with splitted (double) fingers are presented which agree well with measured admittance characteristics.

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